Calculus Curriculum

This curricula and accompanying instructional materials have been developed to align with the NJSLS and in accordance with the NJ Department of Education's guidelines to include: Curriculum designed to meet grade level expectations, integrated accommodations and modifications for students with IEPs, 504s, ELLs, and gifted and talented students, assessments including benchmarks, formative, summative, and alternative assessments, a list of core instructional and supplemental materials, pacing guide, interdisciplinary connections, integration of 21st century skills, integration of technology, and integration of 21st Century Life and Career standards.

About the Standards

In 1996, the New Jersey State Board of Education adopted the state's first set of academic standards called the Core Curriculum Content Standards. The standards described what students should know and be able to do upon completion of a thirteen-year public school education. Over the last twenty years, New Jersey's academic standards have laid the foundation for local district curricula that is used by teachers in their daily lesson plans.

Revised every five years, the standards provide local school districts with clear and specific benchmarks for student achievement in nine content areas. Developed and reviewed by panels of teachers, administrators, parents, students, and representatives from higher education, business, and the community, the standards are influenced by national standards, research-based practice, and student needs. The standards define a "Thorough and Efficient Education" as guaranteed in 1875 by the New Jersey Constitution. Currently the standards are designed to prepare our students for college and careers by emphasizing high-level skills needed for tomorrow's world.

The New Jersey Student Learning Standards include Preschool Teaching and Learning Standards, as well as nine K-12 standards for the following content areas: **21st Century Life and Careers, Comprehensive Health and Physical Education, English Language Arts, Mathematics, Science, Social Studies, Technology, Visual and Performing Arts, World Languages**

The most recent review and revision of the standards occurred in 2014. However, the standards in language arts and math underwent an additional review in 2015 with adoption by the New Jersey State Board of Education in May 2016.

Lower Cape May Regional School District Calculus Curriculum	
Content Area: Mathematics	
Course Title: Calculus	**Grade level: 11/12**
Unit 1: Limits and Their Models	**Dates for Units: 3 weeks**
Unit 2: Differentiation	**Dates for Units: 5 weeks**
Unit 3: Applications of Differentiation	**Dates for Units: 4 weeks**
Unit 4: Integration	**Dates for Units: 3 weeks**
Unit 5: Logarithmic, Exponential, and other Transcendental Functions	**Dates for Units: 5 weeks**
Unit 6: Applications of Integration	**Dates for Units: 2 weeks**
Date Created: 01/24/2020	**Board Approved On: 02/27/2020**

Lower Cape May Regional School District Calculus Curriculum
Unit 1 Overview

Content Area: Mathematics

Unit Title: Limits and Their Models

Target Course/Grade Level: Calculus 11/12

Unit Summary:

- In this unit students will review calculus and compare it to precalculus. Students will discuss limits, graphs and models, functions and their graphs, and will find limits both graphically and numerically. Students will evaluate limits analytically.
- Students will discuss continuity and one sided limits, as well as limits at infinity.

Interdisciplinary Connections:

- **Literacy (using context to identify key vocabulary in word problems), research skills, critical thinking skills, understanding historical context and need/development of math over time.**

21st Century Themes, Skills, and Standards:

- State 21st century themes: Link http://www.state.nj.us/education/cccs/2014/career/
- Technology utilization in the form of PowerPoint presentations, use of Chromebooks, graphing calculators, etc.
- 21st Century Life and Career Standard 9.1, including critical thinking, problem solving, creativity, innovation, collaboration, teamwork and leadership, cross-cultural understanding and interpersonal communication and science.

Learning Targets

CPI #	Cumulative Progress Indicators (CPI) for Unit
(R) N-Q.1.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
(R) N-Q.2.	Define appropriate quantities for the purpose of descriptive modeling.
(R) N-Q.3.	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities

F-IF.1.	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
F-IF.2.	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
F-IF.7.	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F-IF.8.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.
F-IF.9.	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Unit Enduring Questions:	**Unit Enduring Understandings:**
• **What is the definition of Calculus?** • **How does Calculus compare to Pre-calculus?** • **What is a tangent line and how is it basic to Calculus?** • **How can you use the Area Problem?** • **What is a limit?** • **What are two behaviors associated with nonexistent limits?** • **What are the properties of continuity?**	• **Differentiate between a problem that can be solved using Pre- Calculus and Calculus.** • **Interpret mathematical models for real life data** • **Develop and use strategies for finding limits**

• **What is the Intermediate value theorem?** • **What is an asymptote?** • **What are the properties of infinite limits?**	
Unit Objectives: ***Students will know….*** • **How to sketch graphs** • **How to find intercepts** • **How to test for symmetry w/respect to axis and origin** • **Find the points of intersection of two graphs** • **Use function notation** • **Determine continuity at a point and continuity on an open interval** • **Determine one sided limits and continuity on a closed interval** • **Use properties of continuity** • **Understand and use the intermediate value theorem** • **Determine the infinite limits from the left and from the right** • **Find and sketch the vertical asymptote of the graph of a function**	**Unit Objectives:** ***Students will be able to…...*** • **Find domain and range** • **Identify transformations** • **Classify functions** • **Approximate area under a curve** • **Estimate a limit using a numerical or graphical approach** • **Learn the formal definition of a limit** • **Evaluate a limit using properties of limits** • **Develop and use strategies for finding limits** • **Evaluate a limit using dividing out and rationalizing techniques** • **Evaluate a limit using the squeeze Theorem**

Lower Cape May Regional School District Calculus Curriculum Unit 2 Overview
Content Area: Mathematics
Unit Title: Differentiation
Target Course/Grade Level: Calculus 11/12
Unit Summary: ● In this unit students will discuss the derivative and the tangent line problem, they'll explore basic differentiation rules and rates of change, they will discuss product and quotient rules and higher order derivatives, the chain rules, implicit differentiation, and related rates.

Interdisciplinary Connections: Literacy (using context to identify key vocabulary in word problems), research skills, critical thinking skills, understanding historical context and need/development of math over time.

21st Century Themes, Skills, and Standards:

- 21st century themes: Link http://www.state.nj.us/education/cccs/2014/career/
- Technology utilization in the form of PowerP0int presentations, chromebook use, graphic calculators, etc.
- 21st Century Life and Career Standard 9.1, including critical thinking, problem solving, creativity, innovation, collaboration, teamwork and leadership, cross-cultural understanding and interpersonal communication and science.

Learning Targets

CPI #	Cumulative Progress Indicators (CPI) for Unit
(R) N-Q.1.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
(R) N-Q.2.	Define appropriate quantities for the purpose of descriptive modeling
(R) N-Q.3.	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities
(R) A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A-CED.1.	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
A-CED.2.	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
A-CED.4.	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
A-REI.11.	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions

F-IF.1.	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
F-IF.2.	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context
F-IF.3.	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*

Unit Enduring Questions:

- **What is the definition of the derivative of a function?**
- **What are the four notations used for derivatives?**
- **What is continuity and why is it important in differentiation?**
- **What is the relationship between position, velocity and acceleration?**
- **Name some other real life examples that involve rates of change.**
- **What is the difference between explicit and implicit differentiation?**
- **How can the change of one variable depend on the change of another?**
- **Can you give an example?**

Unit Enduring Understandings:

- **Understand the relationship between differentiability and continuity.**
- **Understand how to distinguish between functions written in explicit and implicit form**
- **Understand how to use implicit differentiation to find the derivative of a function**

Unit Objectives:
Students will know….

- **How to Find the slope of the tangent line to a curve at a point**
- **How to Use the limit definition to find the derivative of a function**
- **How to Understand the relationship between differentiability and continuity**
- **How to Find the derivative of a function using**
  - **The constant rule**
  - **The Power rule**
  - **The constant multiple rule**
  - **The sum and difference rules**
  - **How to Find the derivative of**

Unit Objectives:
Students will be able to…...

- **Find the derivative of a function using**
  - **The product rule**
  - **The quotient rule**
- **Find the derivative of a trigonometric function**
- **Find a higher order derivative of a function**
- **Find the derivative of a composite function using the chain rule**
- **Find the derivative of a function using**
  - **The general power rule**
- **Simplify the derivative of a function using algebra**
- **Find the derivative of a trigonometric**

● **the sine function** ● **the cosine function** ● **How to Define position, velocity and acceleration in terms of calculus** ● **How to Use derivative to find rates of change**	**function using the chain rule** • **Distinguish between functions written in explicit and implicit form** • **Use implicit differentiation to find the derivative of a function** • **Find a related rate** • **Use related rates to solve real life problems**

Lower Cape May Regional School District Calculus Curriculum Unit 3 Overview
Content Area: Mathematics
Unit Title: Applications of Differentiation
Target Course/Grade Level: 11-12
Unit Summary: ● Students will be studying extrema on an interval, Rolle's Theorem and the Mean Value Theorem, increasing and decreasing functions and the first derivative test, concavity and the second derivative test, a summary of curve sketching, optimizing problems, and differentials.
Interdisciplinary Connections: ● **Literacy (using context to identify key vocabulary in word problems), research skills, critical thinking skills, understanding historical context and need/development of math over time.**
21st Century Themes, Skills, and Standards: ● 21st century themes: Link http://www.state.nj.us/education/cccs/2014/career/ ● Technology utilization in the form of PowerP0int presentations, chromebook use, graphic calculators, etc. ● 21st Century Life and Career Standard 9.1, including critical thinking, problem solving, creativity, innovation, collaboration, teamwork and leadership, cross-cultural understanding and interpersonal communication and science.

Learning Targets	
CPI #	**Cumulative Progress Indicators (CPI) for Unit**
(R) N-Q.1.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
(R) N-Q.2.	Define appropriate quantities for the purpose of descriptive modeling.
(R) N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities
(R) A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
F-IF.2.	. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context
F-BF.1. (a,b,c)	Write a function that describes a relationship between two quantities.⋆ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time

Unit Enduring Questions:

- **How can you find if a function has a maximum or minimum value?**
- **How can you find the maximum or minimum of a function?**
- **How can you decide if a function is increasing or decreasing?**
- **How can you decide where a function changes direction?**
- **What does a concave up function tell you that that function has?**
- **What is a horizontal Asymptote?**
- **What are some of the tools used to define the sketch of a function?**

Unit Enduring Understandings:

- **Students will be able to name some real life situations when you would wish to determine least, greatest, optimum, minimum, etc.**
- **Understand the definition of extrema of a function on an interval**
- **Understand Relative extrema of a function on an open interval**
- **Find extrema on a closed interval**
- **Understand and use**

-Rolle's Theorem

-The mean value theorem

Unit Objectives: ***Students will know….*** ● **How to Determine intervals on which a function is increasing or decreasing** ● **How to Apply the first derivative test to find relative extrema of a function** ● **How to Determine intervals on which a function is concave up or concave down** ● **How to Find any points of inflection of the graph of a function** ● **How to Apply the second Derivative test to find relative extrema of a function**	**Unit Objectives:** ***Students will be able to…...*** ● **Analyze and sketch the graph of** **-Rational function** **-Radical function** **-Polynomial function** **-Trigonometric function** ● **Solve applied minimum and maximum problems** ● **Understand the concept of a tangent line approximation** ● **Compare the value of the differential, With the actual change in y, Δy** **Find the differential of a function using differentiation formulas**

Lower Cape May Regional School District Calculus Curriculum Unit 4 Overview
Content Area: Mathematics
Unit Title: Integration
Target Course/Grade Level: Calculus 11/12
Unit Summary: ● In this unit students will study Antiderivatives and Indefinite Integration, area, Riemann Sums and Definite Intergrals, The Fundamental Theorem of Calculus, Integration by substitution, and numerical integration and the Trapezoid Rule.
Interdisciplinary Connections: ● **Literacy (using context to identify key vocabulary in word problems), research skills, critical thinking skills, understanding historical context and need/development of math over time.**
21st Century Themes, Skills, and Standards: ● 21st century themes: Link http://www.state.nj.us/education/cccs/2014/career/

- Technology utilization in the form of PowerP0int presentations, chromebook use, graphic calculators, etc.
- 21st Century Life and Career Standard 9.1, including critical thinking, problem solving, creativity, innovation, collaboration, teamwork and leadership, cross-cultural understanding and interpersonal communication and science.

Learning Targets

CPI #	Cumulative Progress Indicators (CPI) for Unit
(R) N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
(R) N-Q.2	Define appropriate quantities for the purpose of descriptive modeling
(R) N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities
(R) A-REI.3.	. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
F-IF.8 (a,b)	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
F-BF.1. (a,b,c)	Write a function that describes a relationship between two quantities.⋆ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
F-BF.3.	. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Unit Enduring Questions: ● **What is an antiderivative?** ● **Why is it called "an" antiderivative instead of "the' antiderivative?** ● **What is sigma notation?**	**Unit Enduring Understandings:** ● **Understand the concept of area** ● **Understand the definition of a Riemann sum** ● **Understand the use of the mean value theorem for integrals**
Unit Objectives: ***Students will know….*** ● **How to Write the general solution of a differential equation** ● **How to Use the indefinite integral notation for antiderivatives** ● **How to Use basic integration rules to find Antiderivatives** ● **How to Find a particular solution of a differential equation** ● **How to Evaluate a definite integral**	**Unit Objectives:** ***Students will be able to…...*** ● **Use sigma notation to write and evaluate a sum** ● **Understand the concept of area** ● **Approximate the area of a plane region** ● **Find the area of a plane region using limits** ● **Evaluate a definite integral (using limits and using properties of definite integrals)** ● **Use pattern recognition to find an indefinite integral** ● **Use change of variables to find and indefinite integral** ● **Use the general power rule for integration to find an indefinite integral** ● **Use a change of variables to evaluate a definite integral** ● **Evaluate a definite integral involving an even or odd function** ● **Approximate a definite integral** **-Using the Trapezoidal rule** **-Using the Simpson's rule** ● **Analyze the approximate errors in the in the Trapezoidal rule and the Simpson's rule**

using the fundamental theorem of calculus ● **How to Understand and use the mean value theorem for integrals** ● **How to Find the average value of a function over a closed interval** ● **How to Understand and use the Second fundamental theorem of calculus**	
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Lower Cape May Regional School District Calculus Curriculum
Unit 5 Overview

Content Area: Mathematics

Unit Title: Logarithmic, Exponential, and Other Transcendental Functions

Target Course/Grade Level: Calculus 11/12

Unit Summary:

In this unit students will learn about: The Natural Logarithmic Function and Differentiation, The Natural Logarithmic Function and Integration, inverse functions, exponential functions, bases other than e and applications, differential equations: growth and decay, differential equations: separation of variables, and inverse trigonometric functions and differentiation.

Interdisciplinary Connections:

- **Literacy (using context to identify key vocabulary in word problems), research skills, critical thinking skills, understanding historical context and need/development of math over time.**

21st Century Themes, Skills, and Standards:

- 21st century themes: Link http://www.state.nj.us/education/cccs/2014/career/
- Technology utilization in the form of PowerP0int presentations, chromebook use, graphic calculators, etc.
- 21st Century Life and Career Standard 9.1, including critical thinking, problem solving, creativity, innovation, collaboration, teamwork and leadership, cross-cultural understanding and interpersonal communication and science.

Learning Targets

CPI #	Cumulative Progress Indicators (CPI) for Unit
(R) N-Q.1.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
(R) N-Q.2.	Define appropriate quantities for the purpose of descriptive modeling.
(R) N-Q.3.	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities
(R) A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
F-BF.4. (a-d)	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠□1. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
F-BF.5.	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents
F-TF.1-4	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F-TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for €/3, €/4 and €/6, and use the unit circle to express the values of sine, cosine, and tangent for €–*x*, €+*x*, and 2€–*x* in terms of their values for *x*, where *x* is any real number. F-TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TF.5-7	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.⋆ F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.⋆

Unit Enduring Questions:

- **What is e and how big is it?**
- **What is the difference between log and ln?**
- **What is an inverse function?**

Unit Enduring Understandings:

- ❖ Develop and use properties of the natural logarithmic function
- ❖ Understand the definition of the number e

Find derivative of functions involving the natural logarithmic function

- ❖ Use the log rule for integral to integrate a rational function

Integrate trigonometric functions

Unit Objectives:
Students will know….

- ❖ How to Verify that one function is the inverse of another function
- ❖ How to Determine whether a function has an inverse function
- ❖ How to find the derivative of an inverse function
- ❖ How to Develop properties of the natural exponential function
- ❖ How to Differentiate natural exponential function
- ❖ How to Integrate natural exponential functions

Unit Objectives:
Students will be able to…...

- ❖ Define exponential functions that have bases other than e
- ❖ Differentiate and integrate exponential functions that have bases other than e.

Use exponential functions to model compound interest and exponential growth

- ❖ Use separation of variables to solve a simple differential equation

Use exponential functions to model growth and decay in applied problems

- ❖ Resolve and solve differential equations that can be solved by separation of variables

Recognize and solve homogeneous differential equations

- ❖ Develop properties of the six inverse trigonometric function
- ❖ Differentiate an inverse trigonometric function

Review the basic differentiation rules for elementary functions

Lower Cape May Regional School District Calculus Curriculum
Unit 6 Overview

Content Area: Mathematics

Unit Title: Applications of Integration

Target Course/Grade Level: Calculus 11/12

Unit Summary:

- In this unit students will study the area of a region between two curves and volume: the disc method.

Interdisciplinary Connections:

- **Literacy (using context to identify key vocabulary in word problems), research skills, critical thinking skills, understanding historical context and need/development of math over time.**

21st Century Themes, Skills, and Standards:

- 21st century themes: Link http://www.state.nj.us/education/cccs/2014/career/
- Technology utilization in the form of PowerP0int presentations, chromebook use, graphic calculators, etc.
- 21st Century Life and Career Standard 9.1, including critical thinking, problem solving, creativity, innovation, collaboration, teamwork and leadership, cross-cultural understanding and interpersonal communication and science.

Learning Targets

CPI #	Cumulative Progress Indicators (CPI) for Unit
(R) N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
(R) N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.
(R) N-Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

R) A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters
F-BF.4. (a-d)	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
F-BF.5.	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
F-TF.1.	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F-TF.2.	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle
F-TF.3.	Use special triangles to determine geometrically the values of sine, cosine, tangent for €/3, €/4 and €/6, and use the unit circle to express the values of sine, cosine, and tangent for €–*x*, €+*x*, and 2€–*x* in terms of their values for *x*, where *x* is any real number
F-TF.4-7	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **Model periodic phenomena with trigonometric functions** F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.⋆ F-TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F-TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.⋆

Unit Enduring Questions: • **What practical application would use the area of a region between two curves?** • **What is the definition of a solid of revolution?** • **How is the washer method an extension of the disc method?**	**Unit Enduring Understandings:** ● **The area of a region between two curves** ● **Volume using the disc method.**
Unit Objectives: ***Students will know….*** ● **How to Find the area of a region between two curves using**	**Unit Objectives:** ***Students will be able to…...*** ● **Find the volume of a solid -of revolution using the disc method**

integration ● **Find the area of a region between intersecting curves using integration**	**-of revolution using the washer method** **-with known cross section**

Lower Cape May Regional School District (Insert Subject/Content Area) Curriculum
Evidence of Learning

Specific Formative Assessments Utilized in Daily Lessons:

- List examples of specific formative assessments to be utilized daily to gauge student comprehension and drive instruction here. Link here for ideas. More ideas and here.
- If you utilization Kahoot, Socrative, quizlet or other online assessment platforms list those here as well.

Summative Assessment Utilized throughout Units:

- QBA's
- Benchmarks
- Formal assessments (test/quiz)
- Exploratory activities (lab)
- Writing journals
- Homework
- Real Life Applications
- Peer Assessment
- Teacher Observations

Modifications for ELL's, Special Education, 504, and Gifted and Talented Students:

- Teacher tutoring
- Peer tutoring
- Cooperative Learning Groups
- Modified Assignments
- Differentiated Instruction
- Response to Intervention (www.help4teachers.com)
- Follow all IEP and 504 modifications

Teacher Notes:

- As required by the NJ Department of Education, teachers in all content areas will integrate the 21st Century Life and Careers Standards. As the NJDOE indicates, "Providing New Jersey students with the life and career skills needed to function optimally within this dynamic context is a critical focus and organizing principle of K-12 public education. New Jersey has both an obligation to prepare its young people to thrive in this environment, and a vested economic interest in grooming an engaged citizenry made up of productive members of a global workforce that rewards innovation, creativity, and adaptation to change." The links below indicate the CPIs for grade ranges and need to be addressed throughout the units of study:

Life and Career Standards

- As indicated in the NJSLS, standards and interdisciplinary connections will be integrated throughout content area curriculum.

Project-based Learning Tasks:

Varies per unit but typically includes exploratory labs and real life applications

Vocabulary:

- In-text vocabulary should be incorporated into every unit. Word journals, vocabulary walls, and/or various other activities should be utilized by the instructor to teach vocabulary.

The Research Process:

- The research process must be integrated within each course curriculum. Student will be provided with opportunities to investigate issues from thematic units of study. As the NJSLS indicate, students will develop proficiency with MLA or APA format as applicable.

Technology:

- Students must engage in technology applications integrated throughout the curriculum. Applicable technology utilized in this curricula are included below:
  -chromebooks
  -graphing calculators
  -projector
  -website based interactive programs

Resources:

Classroom textbook/ Chromebooks

Differentiation Strategies

Differentiation strategies can require varied amounts of preparation time. High-prep strategies often require a teacher to both create multiple pathways to process information/demonstrate learning and to assign students to those pathways. Hence, more ongoing monitoring and assessment is often required. In contrast, low-prep strategies might require a teacher to strategically create process and product choices for students, but students are allowed to choose which option to pursue given their learning profile or readiness level. Also, a low-prep strategy might be focused on a discrete skill (such as vocabulary words), so there are fewer details to consider. Most teachers find that integration of one to two new low-prep strategies and one high-prep strategy each quarter is a reasonable goal.

Low Prep Strategies (add to list as needed)	
Varied journal prompts, spelling or vocabulary lists	Students are given a choice of different journal prompts, spelling lists or vocabulary lists depending on level of proficiency/assessment results.
Anchor activities	Anchor activities provide meaningful options for students when they are not actively engaged in classroom activities (e.g., when they finish early, are waiting for further directions, are stumped, first enter class, or when the teacher is working with other students). Anchors should be directly related to the current learning goals.
Choices of books	Different textbooks or novels (often at different levels) that students are allowed to choose from for content study or for literature circles.
Choices of review activities	Different review or extension activities are made available to students during a specific section of the class (such as at the beginning or end of the period).
Homework options	Students are provided with choices about the assignments they complete as homework. Or, students are directed to specific homework based on student needs.
Student-teacher goal setting	The teacher and student work together to develop individual learning goals for the student.
Flexible grouping	Students might be instructed as a whole group, in small groups of various permutations (homogeneous or heterogeneous by skill or interest), in pairs or individual. Any small groups or pairs change over time based on assessment data.
Varied computer programs	The computer is used as an additional center in the classroom, and students are directed to specific websites or software that allows them to work on skills at their level.
Multiple Intelligence or Learning Style options	Students select activities or are assigned an activity that is designed for learning a specific area of content through their strong intelligence (verbal-linguistic, interpersonal, musical, etc.)
Varying scaffolding of same organizer	Provide graphic organizers that require students to complete various amounts of information. Some will be more filled out (by the teacher) than others.
Think-Pair-Share by readiness, interest, and/or learning profile	Students are placed in predetermined pairs, asked to think about a question for a specific amount of time, then are asked to share their

	answers first with their partner and then with the whole group.
Mini workshops to re-teach or extend skills	A short, specific lesson with a student or group of students that focuses on one area of interest or reinforcement of a specific skill.
Orbitals	Students conduct independent investigations generally lasting 3-6 weeks. The investigations "orbit" or revolve around some facet of the curriculum.
Games to practice mastery of information and skill	Use games as a way to review and reinforce concepts. Include questions and tasks that are on a variety of cognitive levels.
Multiple levels of questions	Teachers vary the sorts of questions posed to different students based on their ability to handle them. Varying questions is an excellent way to build the confidence (and motivation) of students who are reluctant to contribute to class discourse. Note: Most teachers would probably admit that without even thinking about it they tend to address particular types of questions to particular students. In some cases, such tendencies may need to be corrected. (For example, a teacher may be unknowingly addressing all of the more challenging questions to one student, thereby inhibiting other students' learning and fostering class resentment of that student.)
High Prep Strategies (add to list as needed)	
Cubing	Designed to help students think about a topic or idea from many different angles or perspectives. The tasks are placed on the six sides of a cube and use commands that help support thinking (justify, describe, evaluate, connect, etc.). The students complete the task on the side that ends face up, either independently or in homogenous groups.
Tiered assignment/ product	The content and objective are the same, but the process and/or the products that students must create to demonstrate mastery are varied according to the students' readiness level.
Independent studies	Students choose a topic of interest that they are curious about and wants to discover new information on. Research is done from questions developed by the student and/or teacher. The researcher produces a product to share learning with classmates.
4MAT	Teachers plan instruction for each of four learning preferences over the course of several days on a given topic. Some lessons focus on

	mastery, some on understanding, some on personal involvement, and some on synthesis. Each learner has a chance to approach the topic through preferred modes and to strengthen weaker areas
Jigsaw	Students are grouped based on their reading proficiency and each group is given an appropriate text on a specific aspect of a topic (the economic, political and social impact of the Civil War, for example). Students later get into heterogeneous groups to share their findings with their peers, who have read about different areas of study from source texts on their own reading levels. The jigsaw technique allows you to tackle the same subject with all of your students while discreetly providing them the different tools they need to get there.
Multiple texts	The teacher obtains or creates a variety of texts at different reading levels to assign strategically to students.
Alternative assessments	After completing a learning experience via the same content or process, the student may have a choice of products to show what has been learned. This differentiation creates possibilities for students who excel in different modalities over others (verbal versus visual).
Modified Assessments	Assessments can be modified in a variety of ways – for example by formatting the document differently (e.g. more space between questions) or by using different types of questions (matching vs. open ended) or by asking only the truly essential questions.
Learning contracts or Personal Agendas	A contract is a negotiated agreement between teacher and student that may have a mix of requirements and choice based on skills and understandings considered important by the teacher. A personal agenda could be quite similar, as it would list the tasks the teacher wants each student to accomplish in a given day/lesson/unit. Both Learning contracts and personal agendas will likely vary between students within a classroom.
Compacting	This strategy begins with a student assessment to determine level of knowledge or skill already attained (i.e. pretest). Students who demonstrate proficiency before the unit even begins are given the opportunity to work at a higher level (either independently or in a group).
Literature circles	Flexible grouping of students who engage in different studies of a piece of literature. Groups can be heterogeneous and homogeneous.
Learning Centers	A station (or simply a collection of materials) that students might use independently to explore topics or practice skills. Centers allow

	individual or groups of students to work at their own pace. Students are constantly reassessed to determine which centers are appropriate for students at a particular time, and to plan activities at those centers to build the most pressing skills.
Tic-Tac-Toe Choice Board (sometimes called "Think-Tac-Toe"	The tic-tac-toe choice board is a strategy that enables students to choose multiple tasks to practice a skill, or demonstrate and extend understanding of a process or concept. From the board, students choose (or teacher assigns) three adjacent or diagonal. To design a tic-tac-toe board: - Identify the outcomes and instructional focus - Design 9 different tasks - Use assessment data to determine student levels - Arrange the tasks on a tic-tac-toe board either randomly, in rows according to level of difficulty, or you may want to select one critical task to place in the center of the board for all students to complete.
Curriculum development Resources/Instructional Materials:	
List or Link Ancillary Resources and Curriculum Materials Here: NJDOE website	
Board of Education Approved Text(s)	
• Calculus by Anton, Bivens, Davis	